CS 314: Principles of Programming Langauges

Tail Recursion

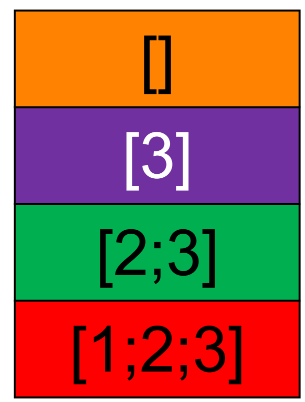
Reverse

```
let rec rev l = match l with
      [] -> []
    | (x::xs) -> (rev xs) @ [x]
```

- Pushes a stack frame on each recursive call

```
rev [1;2;3]
 → (rev [2;3]) @ [1]
 → ((rev [3]) @ [2]) @ [1]
 → (((rev []) @ [3]) @ [2]) @ [1]
 → (([] @ [3]) @ [2]) @ [1]
 → ([3] @ [2]) @ [1]
 → [3;2] @ [1]
 → [3;2;1]
```

Stack: values of `l`



A Clever Version of Reverse

```
let rec rev_helper l a = match l with
    [] -> a
  | (x::xs) -> rev_helper xs (x::a)
let rev l = rev_helper l []
```

- No need to push a frame for each call!

```
rev [1;2;3] →
rev_helper [1;2;3] [] →
rev_helper [2;3] [1] →
rev_helper [3] [2;1] →
rev_helper [] [3;2;1] →
[3;2;1]
```

Stack: values of `l`

[]

Factorial

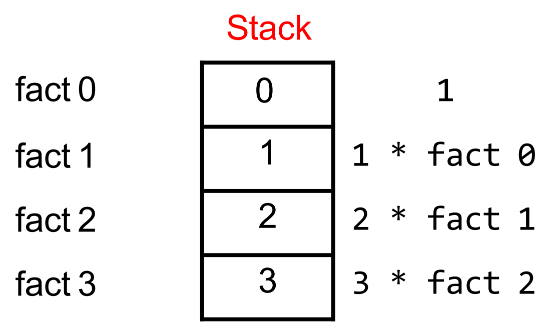
$$\text{fact } n = \begin{cases} 1 & n=0 \\ n * \text{fact } (n-1) & n>0 \end{cases}$$

```
let rec fact  n =
    if n = 0 then 1
    else n * fact (n-1)
;;

fact 4 = 24
```

Factorial

```
fact 3 = 3 * fact 2
               2 * fact 1
                   1 * fact 0
                   1 * 1
                2 * 1
           3 * 2
= 6
```



Stackoverflow?

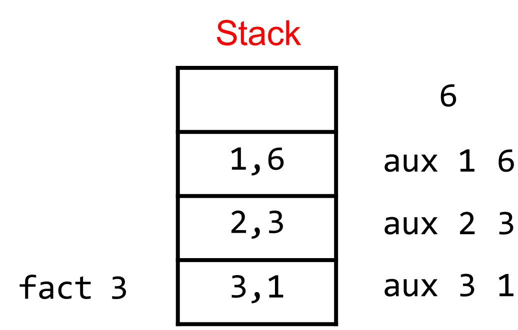
fact 1000000?

```
# let rec fact n = if n = 0 then 1 else n * fact (n-1);;
val fact : int -> int = <fun>
# fact 1000000;;
Stack overflow during evaluation (looping recursion?).
```

Yet Another Factorial

```
let fact n =
  let rec aux x acc =
    if x = 1 then acc
    else aux (x-1) (acc*x)
  in
 aux n 1
```

```
fact 3 = aux 3 1
         aux 2 3
          aux 1 6
         6
```



Tail Recursion

- Whenever a function ends with a recursive call, it is called tail recursive
  - Its “tail” is recursive
- Tail recursive functions can be implemented without requiring a stack frame for each call
  - No intermediate variables need to be saved, so the compiler overwrites them
- Typical pattern is to use an accumulator to build up the result, and return it in the base case

Compare rev and rev_helper

```
let rec rev l =
  match l with
    [] -> []
  | (x::xs) -> (rev xs) @ [x]
```

Waits for recursive call's result to compute final result

```
let rec rev_helper l a =
  match l with
    [] -> a
  | (x::xs) -> rev_helper xs (x::a)
```

final result ***is*** *the result of the recursive call*

Exercise: Finish Tail-recursive Version

```
let rec sumlist l =
  match l with
    [] -> 0
  | (x::xs) -> (sumlist xs) + x
```

Tail-recursive version:

```
let sumlist l =
  let rec helper l a =
    match l with
      [] -> ___
    | (x::xs) -> ___________ in
  helper l 0
```

Exercise: Finish Tail-recursive Version

```
let rec sumlist l =
  match l with
    [] -> 0
  | (x::xs) -> (sumlist xs) + x
```

Tail-recursive version:

```
let sumlist l =
  let rec helper l a =
    match l with
      [] -> a
    | (x::xs) -> helper xs (x+a) in
  helper l 0
```

Quiz #1

True/false: map is tail-recursive.

```
let rec map f = function
  [] -> []
| (h::t) -> (f h)::(map f t)
```

A. True
B. False

Quiz #1

True/false: map is tail-recursive.

```
let rec map f = function
   [] -> []
| (h::t) -> (f h)::(map f t)
```

A. True

B. False

Quiz #2

True/false: fold_left is tail-recursive

```
let rec fold_left f a = function
  [] -> a
| (h::t) -> fold f (f a h) t
```

A. True
B. False

Quiz #2

True/false: fold_left is tail-recursive

```
let rec fold_left f a = function
  [] -> a
| (h::t) -> fold f (f a h) t
```

A. **True**
B. False

Quiz #3

True/false: fold_right is tail-recursive

```
let rec fold_right f l a =
  match l with
    [] -> a
  | (h::t) -> f h (fold_right f t a)
```

A. True
B. False

Quiz #3

True/false: fold_right is tail-recursive

```
let rec fold_right f l a =
  match l with
    [] -> a
  | (h::t) -> f h (fold_right f t a)
```

A. True

B. False

Tail Recursion is Important

- Pushing a call frame for each recursive call when operating on a list is dangerous
  - One stack frame for each list element
  - Big list = stack overflow!

- So: favor tail recursion when inputs could be large (i.e., recursion could be deep). E.g.,
  - Prefer **`List.fold_left`** to **`List.fold_right`**
    - Library documentation should indicate tail recursion, or not
  - Convert recursive functions to be tail recursive

Tail Recursion Pattern (1 argument)

```
let func x =
  let rec helper arg acc =
    if (base case) then acc
    else
      let arg' = (argument to recursive call)
      let acc' = (updated accumulator)
      helper arg' acc' in (* end of helper fun *)
  helper x (initial val of accumulator)
;;
```

Tail Recursion Pattern with fact

```
let fact x =
  let rec helper arg acc =
    if arg = 0 then acc
    else
      let arg’ = arg – 1 in
      let acc’ = acc * arg in
      helper arg’ acc’ in (* end of helper fun *)
  helper x 1
;;
```

Tail Recursion Pattern with rev

```
let rev x =
  let rec rev_helper arg acc =
    match arg with [] -> acc
    | h::t ->
      let arg' = t in
      let acc' = h::acc in
      rev_helper arg' acc' in (* end of helper fun *)
  rev_helper x []
;;
```

Can generalize to more than one argument, and multiple cases for each recursive call

Quiz #4

True/false: this is a tail-recursive map

```
let map f l =
  let rec helper l a =
    match l with
      [] -> a
    | h::t -> helper t ((f h)::a)
  in helper l []
```

A. True
B. False

Quiz #4

True/false: this is a tail-recursive map

```
let map f l =
  let rec helper l a =
    match l with
      [] -> a
    | h::t -> helper t ((f h)::a)
  in helper l []
```

A. True

B. False (elements are reversed)

A Tail Recursive **map**

```
let map f l =
  let rec helper l a =
    match l with
      [] -> a
    | h::t -> helper t ((f h)::a)
  in rev (helper l [])
```

Could instead change **`(f h)::a`** to be **`a@(f h)`**

Q: Why is the above implementation a better choice?

A: $O(n)$ running time, not $O(n^2)$ (where n is length of list)

Outlook: Is Tail Recursion General?

- A function that is tail-recursive returns **at most once** (to its caller) when completely finished
  - The final result is exactly the result of a recursive call; no stack frame needed to remember the current call
- Is it possible to convert an *arbitrary program* into an equivalent one, except where **no call ever returns**?
  - Yes. This is called **continuation-passing style**
  - We will look at this later, if we have time